Review of Spectroscopic Determination of Extra Spatial Dimensions in the
Early Universe

Douglas J. Buettner
Aerospace Corporation
P.O. Box 92957 - M1/112
Los Angeles, CA 90009-2957 [1]

P.D. Morley
General Dynamics Corporation
National Systems Group
14700 Lee Road Chantilly, VA 20151[2]

Ivan Schmidt
Department of Physics
Universidad Técnica Federico Santa María
Casilla 110-V, Valparaíso, Chile[3]

Abstract

Atomic and molecular transitions for high Z objects in the early
universe give bounds to the possible existence of extra spatial dimen-
sions $D = 3 + \epsilon$. We review the theory and present observational data,
based on Lyman and Balmer hydrogen transitions of distant quasars
and CO rotational transitions of far away giant molecular clouds.

[1]e-mail address: Douglas.J.Buettner@aero.org
[2]e-mail address: peter.morley@gd-ais.com
[3]e-mail address: ischmidt@fis.utfsm.cl

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1 Introduction

We live in a four dimensional space-time world. This has been checked experimentally with great precision [1]. Nevertheless, in principle there does not seem to be a reason for this, and in fact the universe might have any number of dimensions.

The physics of extra dimensions began with the work of Kaluza and Klein. They proposed uniting Maxwell's theory of electromagnetism and Einstein's theory of gravitation by embedding them into a generally covariant five-dimensional space-time, whose fifth dimension was curled up into a tiny ring which was not experimentally observable. More complicated non-abelian theories can be obtained in much the same way, by starting with more dimensions and compactifying them in various ways.

In recent years the idea of extra dimensions has been resurrected. The main reason is that the leading candidate for providing a framework in which to build a theory which unifies all interactions, superstrings, has been found to be mathematically consistent only if there are six or seven extra spatial dimensions. Otherwise the theory is anomalous. In conventional string theory the compactification of these extra dimensions occurs at very high scales, close to the Planck scale. Here the extra dimensions manifest themselves mainly through threshold effects of heavy states with masses close to the Planck mass. These models contain the Standard Model (SM) of particle interactions gauge group, and can also accommodate the Minimal Supersymmetric Standard Model. Their main interesting feature is that there is an automatic unification of gauge couplings at a scale M_{un}. Nevertheless, there is still a hierarchy problem because we have no way of explaining why the scales of particle physics are so different from those of gravity.

In the last years there have been several attempts to solve the hierarchy problem by using extra dimensions [2]. If space-time is fundamentally $(4+n)$-dimensional, the 4-dimensional Planck mass $M_{Pl}^{(4)} \simeq 1.22 \times 10^{19} GeV$ depends on the $(4+n)$- dimensional Planck mass M_{Pl} and the volume V_c of the compact extra dimensions through

$$M_{Pl}^{(4)^2} = M_{Pl}^{n+2} V_c.$$

Since no extra dimensions have been detected experimentally, the compactification scale $(\sim 1/V_c^{1/n})$ would have to be much smaller than the weak scale $(100GeV)^{-1}$, and the particles and forces of the SM (except for gravity) must be confined to the four dimensional world volume of a three-brane. By taking V_c large enough it is possible to eliminate the hierarchy problem between the weak and Planck scales.

In all the above scenarios it is essential to know as precise as possible the number of space dimensions. Since at present this number is three [1], and since in the very early universe there exists the possibility of a larger number, then this indicates that it should be possible to define an effective number of space dimensions which changes continuously with time, from its very early universe value to the present one. Moreover, by looking at relics of the early universe, such as the cosmic microwave background or the light emitted by quasars with a very high redshift, it should be possible to measure deviations of the number of spatial dimensions, D, from its present (epoch) value of 3. This review looks at the spectroscopic method of determining $\epsilon = D - 3$ in the early universe.

2   Quantum Mechanical Energy Levels in D-dimensions

The spacing of atomic and molecular energy levels varies with the dimension of space. An intuitive way to understand this is the following. Both the kinetic energy term and the Coulomb potential in the Schrödinger equation are related to the Laplacian operator. This operator has the property of measuring the difference between the average value of the field in the immediate neighborhood of a point and the precise value of the field at the point. How the field spreads out from the point depends on the dimension of space it resides in. In particular, ϵ differences between two fractal geometries give rise to corresponding ϵ differences in their Laplacians.

Under some circumstances, it is possible to solve the Schrödinger equation in $D = 3 + \epsilon$ dimension of space ($4 + \epsilon$ dimension of space-time), using a Taylor expansion [3] about $D = 3$

$$< |E| > |_{D=3+\epsilon} = < |E| > |_{D=3} + \frac{d < |E| >}{dD}|_{D=3}\epsilon + \cdots . \qquad (1)$$

The Hellmann-Feynman theorem is

$$\frac{d < |E| >}{dD}|_{D=3} = < |\frac{\partial H}{\partial D}|_{D=3}| > \qquad (2)$$

where H is the D-dimensional Hamiltonian. In the succeeding sections, we will give the quantum mechanical hydrogen transitions and the linear molecular rotational energies in D-dimensions, but first we turn to the discussion of the technique needed to measure ϵ.

3 How to Determine ϵ

Using ancient light, we are faced with the problem of disentangling the cosmological redshift to obtain the light's true rest-frame wavelength, λ_{epoch}, associated with the earlier time-epoch. In order to do this, we use the fact that all light from the same astronomical object has the same cosmological Z_c:

$$Z_c = (\lambda_{observed} - \lambda_{epoch})/\lambda_{epoch} . \qquad (3)$$

In this equation, we are trying to deduce λ_{epoch} to see if $\lambda_{epoch} \neq \lambda_{present}$. Clearly, an observation of a single line, even if that line can be identified (e.g. Ly$_\alpha$), cannot determine λ_{epoch} because the value of Z_c is unknown. The problem is solvable if we have *two* lines from the same object, which then allows us to uniquely determine λ_{epoch}, and thus ϵ:

$$\epsilon = \frac{\tau_M \lambda_1 - \lambda_0}{a - b\tau_M} \qquad (4)$$

where

$$\tau_M = \frac{\lambda_{0M}}{\lambda_{1M}} \qquad (5)$$

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with λ_{0M}, λ_{1M} the two measured redshifted lines, which have early universe epoch rest-frame wavelengths $\lambda_0 + a\epsilon$, $\lambda_1 + b\epsilon$, where λ_0, λ_1 are the present epoch (i.e. laboratory) transition wavelengths. In this equation, a and b are determined by solving the Schrödinger equation in $D = 3 + \epsilon$ dimensions.

Thus the game is to find observational data of high quality that has at least two identifiable transitions from the same deep-space emitter.

4   Rotational Energy Levels in D-dimensions

We are interested in linear molecules, such as CO, because, being the simplest molecules, they are the ones most likely to be identified in distant, giant molecular clouds. Also, it turns out that the Schrödinger equation in $D = 3 + \epsilon$ dimensions is solvable in this case.

The Hamiltonian is

$$H_{rot} = B(L)L^2 \tag{6}$$

where L is the body (molecule)-fixed rotational angular momentum and B is the principal rotational constant (equal to 1/2I, where I is the principal moment of inertia). Because of centrifugal stretching, $B = B(L)$. We need to generalize this energy to $D = 3 + \epsilon$ fractal space. In quantum mechanics, L^2 is a second order Casimir invariant operator, C_2. Thus the generalized rotational operator is

$$H_{rot} = B(C_2)C_2 \tag{7}$$

The piece $B(C_2)$ has a simple form when no vibrations are excited [4]

$$B(C_2) = B_0 + B_1 C_2 . \tag{8}$$

In general, the second order Casimir invariant is [5]:

$$C_2 = f^i_{jk} f^j_{il} X^k X^l = H_i G_{ij} H_j + \sum_{\text{all roots}} E^\alpha E_{-\alpha} \tag{9}$$

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where f^i_{jk} are the structure constants, X_k are generators, and C_2 commutes with all generators. The Racah formula for the eigenvalue of C_2 for any irreducible representation is easily derived by letting C_2 act on the state with highest weight Λ. The result is:

$$C_2 = (\Lambda, \Lambda + 2\delta) \qquad (10)$$

where $\delta = (1, 1, ..., 1)$ in the Dynkin basis. The scalar product of any two weights can be written as:

$$(\Lambda, \Lambda') = \sum_{ij} a'_j G_{ij} a_j \qquad (11)$$

where G_{ij} is a symmetric tensor whose elements can be computed for each simple group, and which are given in Table 7 of [5], and the a_i are the Dynkin components of Λ.

In our case we want to find the Casimir invariant for the $(L, 0, ...0)$ totally symmetric representation. The choice of representation depends, of course, on the way in which we want to generalize angular momentum, and the correct choice, we argue, would preserve the symmetry properties (in this case this would mean to keep the completely symmetric coupling) when generalizing to larger dimensions.

For odd space dimensions $D = 2n + 1, n = 1, 2, ...$, the algebra is B_n, and for even space dimensions $D = 2n, n = 1, 2, ...$, it is D_n. We calculate C_2, using the equations above, and the a_j values given by $(L, 0, ...0)$:

$$C_2 = (L, 0, ..., 0) G(B_n \text{ or } D_n)(2 + L, 2, 2, ..., 2) \qquad (12)$$

which gives, by simple matrix multiplication, both for B_n and D_n:

$$C_2 = L(L + D - 2) . \qquad (13)$$

The pure (no vibrational quanta) rotational energies in D-dimension space are then:

$$\mathcal{H}_{rot} = [B_0 + B_1(L(L+1))]L(L+1) + \{2B_1 L^2(L+1) + B_0 L\}\epsilon . \qquad (14)$$

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ϵ Transitions	
Lyman line	wavelength
Ly$_\alpha$	$1215.67 + 1418.27\epsilon$
Ly$_\beta$	$1025.72 + 1111.18\epsilon$
Ly$_\gamma$	$972.537 + 1021.155\epsilon$
Ly$_\delta$	$949.743 + 981.391\epsilon$
Ly$_\epsilon$	$937.803 + 960.122\epsilon$

Table 1: ϵ Lyman hydrogen quantum mechanical wavelengths (Ångstroms)

5 Hydrogen Lyman Transitions

The hydrogen atom is the only other system that is amendable to solution in $D = 3 + \epsilon$ dimensions. By means of the Taylor expansion, we obtain in Table 1 the Lyman transitions as a function of ϵ, for $\epsilon \ll 1$. If $\epsilon \neq 0$, the result is a change in each transition by a unique amount. The effect is unmistakable: even a small shift $\epsilon \sim 0.03$ will be detectable.

Lyman data has been analyzed in reference [3]. Except for anomalous data associated with one data set @ $Z_c \sim 4$, $< \epsilon > \approx 0$. High Z_c quasars are extremely rare, but the Sloan Digital Sky Survey (http://www.sdss.org) has discovered several $Z_c \geq 5$. Looking at each Sloan spectrogram, one has difficulty in identifying the original center-line of the Lyman series. The spectrum is a superposition of a priori unknown set of cosmological red shifts. Even the largest or primary shift introduces a complicated transposed spectrum. Next, uneven absorption of radiation as it moves through one intergalactic cloud to another can remove completely the center of a spectral line or one of its wings and may lead to complete masking of the location of the original radiated line. Lastly, line overlaps, which are such a rare occurrence in terrestrial plasmas is the norm for high Z_c spectra. These circumstances have lead to the observation that no astronomy group has been able to locate the matching (same cloud or put in another way, identical Z_c values) Lyman series Ly$_\alpha$ to Ly$_\epsilon$ in one $Z_c \geq 4$ spectrogram. Due to absorption, the Lyman series in hydrogen becomes difficult to use for $< \epsilon >$ determination for very high $Z_c \geq 4$.

ϵ Transitions	
Balmer line	wavelength
H_α	$6562.8 + 4155.24\epsilon$
H_β	$4861.36 + 2834.99\epsilon$
H_γ	$4340.49 + 2417.58\epsilon$

Table 2: ϵ Balmer hydrogen quantum mechanical wavelengths (Ångstroms)

6   Hydrogen Balmer Transitions

The Balmer lines start off in the optical (for small $\epsilon \ll 1$) and get redshifted to the infrared for high Z_c. By using the atmospheric infrared windows where absorption is small, the infrared Balmer lines can evade the difficulties that potentially plague the Lyman series. In Table 2, we give the hydrogen Balmer epsilon-dependent transitions. The measured redshift of $(Z_c + 1)\lambda_0$ means, for example, a $Z_c = 3$ quasar has Balmer lines in the infrared.

Various online databases were searched for papers containing emission spectra from high Z_c quasars that contained coincident hydrogen Balmer lines. Of the many references identified in these databases, only one had tabular data [6]. Another, [7], had a single Balmer line pair measured. In the future, infrared data will be more plentiful as various observatories bring on-line sophisticated infrared spectrometers. We use eq(4) above to determine ϵ. The uncertainty in ϵ, $\delta\epsilon$, is determined by the equation in reference [3].

It is seen that $< \epsilon > \simeq 0$ is favored by this small sample of $Z_c \sim 3$ Balmer data. Balmer data for $Z_c \simeq 4$ would be especially interesting.

7   Molecular Rotational Transitions

The Balmer quasar spectra allow a better determination of $< \epsilon >$ than the Lyman data for high Z_c objects, but the best spectroscopic data would be lines starting off already in the microwave, and redshifted towards the radio, namely rotational spectra.

Balmer H_α and H_β					
Quasar	transition	wavelength (Å)	FWHM (km/s)	FWHM (Å)	σ
Q0007-000	H_α	21572.1	4500	323.8	116.6
	H_β	15896.5	3500	185.6	66.9
Q0027+018	H_α	21952.7	4500	329.5	118.7
	H_β	16091	4500	241.5	87.0
Q0237-233	H_α	21152.1	7500	529.2	190.6
	H_β	15702.1	3000	157.1	56.6
Q1623-268	H_α	23258.7	2600	201.7	72.7
	H_β	17063.3	2500	142.3	51.3
Q1816+475	H_α	21132.4	2600	183.3	66.0
	H_β	15653.5	2600	135.8	48.9

Table 3: Taken from reference [6]

Balmer H_α and H_β			
Quasar	transition	wavelength (Ångstroms)	σ
1937-101	H_α	23228.2	57.2
	H_β	20708.6	64.2

Table 4: Taken from reference [7]

ϵ values		
Quasar	ϵ	$\Delta\epsilon$
Q0007-000	0.11	0.15
Q0027+018	0.24	0.2
Q0237-233	-0.04	0.18
Q1623-268	0.22	0.12
Q1816+475	0	0.09
1937-101	0.059	0.16

Table 5: ϵ from Balmer spectra

$C^{12}O^{16}$ Transitions	
transition	MHz
$1 \rightarrow 0$	115271.202
$2 \rightarrow 1$	230538
$3 \rightarrow 2$	345795.991
$4 \rightarrow 3$	461040.77
$5 \rightarrow 4$	576267.922
$6 \rightarrow 5$	691473.09
$7 \rightarrow 6$	806651.719

Table 6: Laboratory CO pure rotational transitions

Giant Molecular Clouds		
identification	observed transitions	source
QSO J114816.64+525150.3	$7 \rightarrow 6, 6 \rightarrow 5, 3 \rightarrow 2$	[9]
QSO H1413+117	$5 \rightarrow 4, 4 \rightarrow 3, 3 \rightarrow 2$	[10]
QSO PSS 2322+1944	$5 \rightarrow 4, 4 \rightarrow 3, 2 \rightarrow 1, 1 \rightarrow 0$	[11]
QSO BR1202-0725	$7 \rightarrow 6, 5 \rightarrow 4$	[12]
QSO F10214+4724	$6 \rightarrow 5, 3 \rightarrow 2$	[12]
QSO HR10 (J164502+4626.4)	$5 \rightarrow 4, 2 \rightarrow 1$	[13]

Table 7: Giant Molecular Cloud Transitions

The linear molecule CO ($C^{12}O^{16}$) is the main non-hydrogen emitter in distant, giant molecular clouds. In Table 6, we give its (present epoch) laboratory transition frequencies [8]. It is a simple matter to determine the constants that appear in eq(8): $B_0 = 57.635968$ GHz and $B_1 = -1.835 \times 10^{-4}$ GHz for CO. By doing a literature search, we have identified 6 tabulations of CO data that are usable, including data from the presently known farthest quasar J114816.64+525150.3 @ $Z_c = 6.42$.

Data					
QSO	approx Z_c	transition	obs (Ghz)	FWHM	channel width
J114816.64+525150.3	6.42	$7 \rightarrow 6$	108.729	279 km/s	5 MHz
		$6 \rightarrow 5$	93.204	279 km/s	5 MHz
		$3 \rightarrow 2$	46.61	320 km/s	50 MHz
H1413+117	2.56	$5 \rightarrow 4$	161.964	398 km/s	512 MHz
		$4 \rightarrow 3$	129.576	375 km/s	512 MHz
		$3 \rightarrow 2$	97.199	362 km/s	512 MHz
PSS 2322+1944	4.12	$5 \rightarrow 4$	112.55	273 km/s	35 MHz
		$4 \rightarrow 3$	90.05	375 km/s	35 MHz
		$2 \rightarrow 1$	45.035	200 km/s	6.25 MHz
		$1 \rightarrow 0$	22.515	200 km/s	50 MHz
BR1202-0725	4.71	$7 \rightarrow 6$	141.2	300 km/s	equivalent 60 km/s
		$5 \rightarrow 4$	101.3	350 km/s	equivalent 60 km/s
F10214+4724	2.29	$6 \rightarrow 5$	210.5	300 km/s	equivalent 80 km/s
		$3 \rightarrow 2$	105.2	220 km/s	equivalent 80 km/s
HR10	1.44	$5 \rightarrow 4$	235.982	380 km/s	equivalent 75 km/s
		$2 \rightarrow 1$	94.405	400 km/s	equivalent 50 km/s

Table 8: Early Universe CO cloud data

	Processed Data		
QSO	line pairs	ϵ	$\Delta\epsilon$
J114816.64+525150.3	$7 \rightarrow 6, 6 \rightarrow 5$	-0.000012043	0.118937801
	$7 \rightarrow 6, 3 \rightarrow 2$	-0.000004379	0.084521824
	$6 \rightarrow 5, 3 \rightarrow 2$	-0.000003283	0.096631339
H1413+117	$5 \rightarrow 4, 4 \rightarrow 3$	-8.1128E-04	7.2375E-01
	$5 \rightarrow 4, 3 \rightarrow 2$	1.6508E-03	3.2882E-01
	$4 \rightarrow 3, 3 \rightarrow 2$	3.1291E-03	5.6347E-01
PSS 2322+1944	$5 \rightarrow 4, 4 \rightarrow 3$	2.1489E-03	1.1919E-01
	$5 \rightarrow 4, 2 \rightarrow 1$	1.3294E-03	1.3841E-02
	$5 \rightarrow 4, 1 \rightarrow 0$	1.7313E-04	2.8838E-02
	$4 \rightarrow 3, 2 \rightarrow 1$	1.1654E-03	2.0347E-02
	$4 \rightarrow 3, 1 \rightarrow 0$	4.1410E-05	3.1010E-02
	$2 \rightarrow 1, 1 \rightarrow 0$	-5.2042E-04	4.5728E-02
BR1202-0725	$7 \rightarrow 6, 5 \rightarrow 4$	0.149826	0.0765425
F10214+4724	$6 \rightarrow 5, 3 \rightarrow 2$	-0.00775587	0.0210779
HR10	$5 \rightarrow 4, 2 \rightarrow 1$	-3.00361E-05	0.0132079

Table 9: ϵ from CO spectra

8   Conclusion

We take the high resolution data @ Z = 6.42 and perform the statistical Z-test [14] by taking $< \Delta\epsilon >$ as the standard deviation. This predicts that the probability of $\epsilon \neq 0$ is 1 in 7794, only 850 million years (using the standard cosmology) after the Big Bang.

The experimental spectroscopic data from ancient light shows that the dimension of space was 3 (present value) very soon after the Big Bang. The extra dimensions that some theories predict must either occur at very early time or somehow be restricted such that ordinary baryonic matter cannot couple to it.

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